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Fragmentation of the Primordial Gas Clouds and the Lower Limit on the Mass of the First Stars

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ABSTRACT

We discuss the fragmentation of primordial gas clouds in the universe after decoupling. Comparing the time scale of collapse with that of fragmentation, we obtain the typical mass of a fragment both numerically and analytically. It is shown that the estimated mass gives the minimum mass of a fragment which is formed from the primordial gas cloud and is essentially determined by the *Chandrasekhar mass*.

Subject headings: galaxies: formation — molecular processes — stars: formation

1. Introduction

The first stars formed after decoupling had no metals heavier than lithium, so that star formation proceeded without grains. This means that the isothermal approximation, which is often used to investigate star formation in the present molecular cloud (its validity is explained by Hayashi & Nakano 1965), is not applicable and that star formation in the primordial gas cloud may be completely different. Up to now many authors have studied dynamical and thermal evolutions of primordial gas clouds (Matsuda, Sato, & Takeda 1969; Yoneyama 1972; Hutchins 1976; Yoshii & Sabano 1979; Carlberg 1981; Palla, Salpeter, & Stahler 1983; Lahav 1986; Puy & Signore 1996; Susa, Uehara, & Nishi 1996, hereafter paper I). One of the most important quantity in such studies is the minimum Jeans mass in

the primordial gas cloud in relation to the initial mass function. In a spherical free-falling cloud, however, the density perturbation grows only in a power law (Hunter 1964) as in the expanding universe, so that the minimum Jeans mass does not necessarily mean the mass of the fragment (Larson 1985 and references therein). In general a collapsing cloud does not necessarily fragment at the instant when the Jeans mass reaches the minimum, but fragments when the collapse of the cloud is almost halted (see §3).

It is known that a spherical cloud in pressure-free collapse is unstable against non-spherical perturbations (Lin, Mestel, & Shu 1965; Hutchins 1976; paper I). This means that the collapsing primordial gas cloud is not spherical even without angular momentum in general. Therefore the evolution of a non-spherical cloud must be considered. In one of the plausible scenarios, shock waves occur in the collapsing non-spherical primordial gas cloud. In the post shock flow, due to cooling by hydrogen molecules formed through non-equilibrium recombination, a cooled sheet ($T \lesssim 500$K) is formed (MacLow & Shull 1986; Shapiro & Kang 1987, hereafter SK). This cooled sheet fragments more easily into filaments (Miyama, Narita, & Hayashi 1987a, 1987b) than into spherical clouds. As the virial temperature of a filament is essentially *constant* (see eq. [11]), its evolution is much different from that of a spherical cloud whose virial temperature increases as the cloud contracts.

In this Letter we investigate the dynamical and thermal evolution of an infinitely long cylinder to model a filament formed in the post shock cooled sheet and estimate the minimum mass of the fragment of the cylinder. The formulation of our model is shown in §2. In §3 we show how to estimate the mass of the fragment in our model. The numerical results are shown in §4. We analytically estimate the final fragment mass in §5. §6 will be devoted to discussions .

2. Basic equations

2.1. Equation of motion of the cloud for cylindrical collapse

For an isothermal cylinder there is a critical line density $M_c(T)$ (Ostriker 1964)given by

$$M_c(T) = \frac{2k_B T}{\mu m_{\rm H} G}, \tag{1}$$

where T, μ, and $m_{\rm H}$ represent the isothermal temperature, the mean molecular weight and the mass of hydrogen atoms, respectively. An isothermal cylinder with line density M larger than $M_c(T)$ has no equilibrium and collapses, while an isothermal cylinder with M less than $M_c(T)$ expands if the external pressure does not exist.

When the cylindrical cloud is formed as a result of the fragmentation of the sheet, the gas pressure is comparable to the gravitational force. So the effect of the gas pressure must be included in the equation of motion of the cylindrical cloud. To know the dynamical evolution of such a cylinder we begin with the virial equation for a cylinder,

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2T + 2\mathit{\Pi} - GM^2, \tag{2}$$

where

$$I = \int_V \rho r^2 dV, \quad T = \int_V \frac{1}{2}\rho u^2 dV, \quad \mathit{\Pi} = \int_V p dV, \tag{3}$$

and the integration is effected over the volume per unit length of the cylinder, V. Here we approximate that the cylinder is uniform and has constant density ρ, scale radius $R = \sqrt{M/\pi\rho}$, and temperature T. Then above I, T, and $\mathit{\Pi}$ become

$$I = \frac{1}{2}MR^2, \quad T = \frac{1}{4}M\left(\frac{dR}{dt}\right)^2, \quad \mathit{\Pi} = \frac{k_B T}{\mu m_{\rm H}}M \ . \tag{4}$$

Substituting (4) in equation (2) we obtain

$$\frac{d^2R}{dt^2} \quad = \quad -\frac{2G}{R}(M - M_c(T)) \ . \tag{5}$$

We describe the evolution of the cylinder by this equation.

2.2. Energy equation and chemical reactions

The equations describing thermal processes are the same as those of paper I. The time evolution of the cloud temperature is described by the energy equation given by

$$\frac{d\varepsilon}{dt} = -P\frac{d}{dt}\frac{1}{\rho} - \Lambda_{rad} - \Lambda_{chem} \ , \tag{6}$$

where ε, Λ_{rad} and Λ_{chem} are the thermal energy per unit mass , the cooling by the radiation and the cooling/heating by chemical reactions, respectively. In Λ_{rad} we take into account H_2 line emissions due to the vibrational and rotational transitions between excited levels. The gas pressure P is evaluated by the equation of state of the ideal gas. The details of Λ_{rad} are shown in Appendix B of paper I and Λ_{chem} is shown in SK.

We use the same chemical reaction rates as Palla et al. (1983). We consider $\rm H, H_2, H^+, H^-,$ and $\rm e^-$ as chemical compositions of the cloud. Helium and its ions are not included in our calculations because they are chemically inert at such low temperatures ($T \lesssim 10^4$K) as considered here. Hereafter we use f_i to denote the fractional number abundance of the i-th composition.

3. The epoch of fragmentation and the mass of a fragment

The dispersion relation of the linear density perturbations of an isothermal sheet with surface density Σ shows that the perturbation with wavelength $\lambda_{frag} \sim 4\pi H$ grows fastest, where $H = c_s^2/\pi G\Sigma$ and $c_s = \sqrt{k_B T/\mu m_{\rm H}}$ is the sound velocity (Simon 1965). Although the primordial gas clouds we consider are not isothermal, the dispersion relations are weakly dependent on equation of state (see Fig.1 of Larson 1985) and we use the isothermal dispersion relations as typical ones. As shown by Miyama et al. (1987a, b), an equilibrium sheet tends to fragment into filaments, whose line density is estimated by

$$M = \lambda_{frag}\Sigma \sim \frac{4k_B T}{\mu m_{\rm H} G}. \tag{7}$$

Since the above M is greater than $M_c(T)$ the filament formed by fragmentation of an isothermal sheet cannot be in an equilibrium configuration but begins to collapse. As shown by Inutsuka & Miyama (1992) such a cylinder with $M > M_c(T)$ does not fragment immediately. To estimate the epoch of the fragmentation of a collapsing cloud we should compare the dynamical time scale of collapse defined by $t_{dyn} = \rho/\frac{d\rho}{dt}$ with that of fragmentation, t_{frag}. When $t_{dyn} \lesssim t_{frag}$, the cylinder collapses so fast that the density perturbation does not grow similar to the spherical case. The collapsing cloud fragments when the condition

$$t_{dyn} \sim t_{frag}, \tag{8}$$

is satisfied, i.e. when the collapse almost stops. In such a situation we can apply the perturbation analysis of the equilibrium sheet and the cylinder.

In the case of the isothermal cylinder in equilibrium, the perturbation with wavelength $\lambda_{frag} \sim 2\pi R$ grows fastest, where R is the scale radius of the cylinder defined by $R = \sqrt{M/\pi\rho_0}$ with M and ρ_0 being the line density and the central density, respectively. The e-folding time scale of the fragmentation is given by $t_{frag} = 2.1/\sqrt{2\pi G\rho_0}$, (Nagasawa 1987). The mass of the fragment formed by the fragmentation of the isothermal cylinder is estimated by

$$M_{frag} = \lambda_{frag} M \sim 2\pi R M. \tag{9}$$

In our model we estimate the mass of the fragment by equation (9) at the epoch when the condition (8) is satisfied.

4. The results of numerical calculations

To obtain initial conditions of a cylindrical cloud which is formed by the fragmentation of the post shock cooled sheet, we recalculate a typical case of SK with $z = 5, \Omega_b h^2 = 0.1$, and shock velocity $v_s = 200\mathrm{km\,s^{-1}}$. As initial conditions of our cylindrical cloud we adopt physical parameters of the post shock one dimensional flow at the point when the cooling time of the sheet ($\sim$ the dynamical time scale of the sheet) becomes equal to the time scale of the fragmentation, so that the sheet is expected to fragment into filaments. The initial condition of the cylinder thus obtained is $n_0 = 6.9\mathrm{cm^{-3}}, T_0 = 330\mathrm{K},\ f_{\mathrm{H_2},0} = 2.3\times10^{-3}$, and $f_{\mathrm{e^-},0} = 2.7\times10^{-4}$. Under the above initial condition we calculated for the cases $M/M_c(T_0) = 1.0,\ 1.3,\ 1.5,\ 1.7$, and 2.0 (case A$\sim$E, see Table 1), for comparison.

Since the equilibrium sheet tends to fragment into the cylinder of line density $M \sim 2M_c(T_0)$ (eq. [1, 7]), we show the evolution for case E as representative, for which the mass of the fragment becomes minimum. The evolution on the n-T (number density - temperature) plane and virial temperature (eq. [11]) are shown in Figure 1a. From Figure 1a we can see that the cylinder continues to collapse even after the cloud becomes optically thick to the line emission of hydrogen molecules ($n \gtrsim 10^{10}\mathrm{cm^{-3}}$). The fact that the cloud can collapse isothermally as a function of time by radiative cooling ($n \gtrsim 10^{12}\mathrm{cm^{-3}}$) is the characteristic feature of the cylindrical shape of the cloud, for which the gravitational force is proportional to R^{-1}. Since temperature becomes constant, hydrogen molecules does not dissociate. Figure 1b shows two time scales t_{frag} and t_{dyn} normalized by the initial free-fall time scale ($1/\sqrt{2\pi G\rho_0}$) and the mass of the fragment (eq. [9]). Comparing two time scales, we conclude that the collapsing cylinder fragments at $n \sim 3\times10^{12}\mathrm{cm^{-3}}$ and the mass of the fragment becomes $\sim 2M_\odot$ for case E.

In Table 1 number density n, hydrogen molecule fraction $f_{\mathrm{H_2}}$ when the fragmentation occurs (column 3 and 4, respectively), and the mass of the fragment M_{frag} (column 5) are listed for each line densities M (column 2). From Table 1 we can see the tendency that the larger the initial line density M, the smaller the mass of the fragment M_{frag} and M_{frag} converges to several solar mass for larger line densities. Interpretation of these results will be stated in the next section.

5. Analytical estimate of the minimum fragment mass

It is interesting to ask what determines the mass of the fragments of the collapsing cylinder for case E. The collapsing cylinder fragments when the condition (8) is satisfied.

As the cylinder collapses isothermally as a function of time finally, we can set $d\varepsilon/dt \sim 0$ in equation (6) and from this we obtain $t_{dyn} \sim (\gamma - 1)\, t_{cool}$, where $\gamma = 5/2$ for diatomic molecular gas. Since the cloud cools by the line emissions of hydrogen molecules and is optically thick to these lines emission, t_{cool} can be estimated by

$$t_{cool} \sim \frac{\frac{1}{\gamma-1}\frac{M}{\mu m_{\rm H}} k_B T}{2\pi R \sigma T^4 \frac{\Delta\nu}{\nu} \alpha_c}, \tag{10}$$

where $\sigma = 2\pi^5 k_B^4/15h^3c^2$ is the Stefan-Boltzmann constant, and α_c is the effective number of line emissions. Since the optical depth is ~ 100 at most, the wing of line profile may not be important (Rybicki & Lightman 1979) and the line broadening is determined by Doppler broadening as $\Delta\nu/\nu = v_{\rm H_2}/c = \sqrt{k_B T/m_{\rm H} c^2}$. The effects of chemical heating/cooling are not included in t_{cool} because the fraction of hydrogen molecule $f_{\rm H_2}$ is almost unity by the epoch of the fragmentation. The temperature of the cylinder is estimated by the virial temperature as

$$k_B T = \frac{1}{2} \mu m_{\rm H} G M. \tag{11}$$

From the above equations we obtain

$$M_{frag} \sim 2\pi R M \sim \sqrt{\frac{1}{\alpha_c}} \frac{1}{\mu^{9/4}} \frac{m_{Pl}^3}{m_{\rm H}^2}, \tag{12}$$

where $m_{Pl} = \sqrt{hc/G}$ is the Planck mass. Equation (12) means that M_{frag} is essentially equal to the Chandrasekhar mass ($\sim m_{Pl}^3/m_p^2$, here m_p is proton mass).

So far we did not consider other heating processes such as shocks and turbulence which likely to occur in the collapsing cloud. All these processes tend to halt the collapse, so that the epoch of fragmentation becomes earlier and, as a result, the mass of fragment increases. So equation (12) gives the lower limit on the mass of the fragment which is formed from the primordial gas cloud. Our estimate shows even the primordial gas cloud which cools most effectively by hydrogen molecules cannot fragment into a smaller mass than equation (12).

For smaller line densities the gas pressure force halts the collapse before three-body reactions (Palla et al. 1983) *completely* convert atomic hydrogens to molecules. In these cases equation (12) is not applicable because clouds are optically thin to line emissions for $f_{\rm H_2} \lesssim 10^{-1}$ and chemical heating by $\rm H_2$ formation must be considered when hydrogen molecules are formed by three-body reactions. These clouds fragment at lower density, in other words at larger scale radius, than the density at which they reach the isothermally collapsing stage (corresponding to $n \gtrsim 10^{12}{\rm cm}^{-3}$ of case E). Since M_{frag} is proportional to the scale radius at fragmentation, this leads to the larger fragment mass.

6. Discussions

The opacity-limited hierarchical fragmentation scenario was argued by several authors (Low & Lynden-Bell 1976; Rees 1976; Silk 1977). According to Rees (1976), the minimum Jeans mass of the spherical cloud is given by

$$M_F \;=\; \left(\frac{k_B T}{m_{\rm H} c^2}\right)^{1/4} f^{-1/2} \frac{1}{\mu^{9/4}} \frac{m_{Pl}^3}{m_{\rm H}^2},$$

where f ($\lesssim 1$) is the efficiency of the radiation of the cloud compared with the black body radiation and it depends on the details of the cooling process and the opacity. As the present molecular clouds have various molecules and grains, f is nearly equal to unity when the cloud becomes optically thick to the radiation. In the case of the primordial gas cloud which is cooled by line emissions of hydrogen molecules, f is proportional to $\sqrt{T}$ (Doppler broadening) and T-dependence of M_F completely vanishes. Apparently this gives the same conclusion as ours. However previous authors argued the spherical collapse, which does not lead to fragmentation, and our argument would be more relevant.

Although our estimate is order of magnitude and the exact mass of the fragment cannot be estimated, the existence of the lower limit on the mass of the fragment we found would have significant meanings. For example, the following scenario for formation of our Galaxy may be possible: the existence of lower limit on the fragment mass cause the mass of the first stars to be massive, so that the first stars complete main-sequence stage within the age of the universe and evolve to dark remnants (e.g. white dwarfs, neutron stars, and black holes) in the Galactic halo after some mass loss, whose gas fall to the Galactic disk to form it. This scenario would be consistent with infall model (Lynden-Bell 1975; Vader & de Jong 1981; Clayton 1988), which was proposed to explain G dwarf problem (the observed paucity of metal-poor stars, see Rocha-Pinto & Maciel 1996 for recent observation). Remnants thus formed by the above scenario may be MACHOs, whose estimated mass ($\sim 0.5 M_\odot$, Bennett et al.1996) is consistent with that of white dwarfs. It is very interesting that some of astrophysical problems, including non-existence of zero metal stars, may be explained in a consistent way by such a simple scenario.

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Table 1. initial line density and results of numerical calculations

case	$M/M_c(T_0)$	$n(\mathrm{cm}^{-3})$	$f_{\mathrm{H_2}}$	$M_{frag}/M_{\odot}$
A	1.0	1×10^{8}	5×10^{-3}	1×10^{2}
B	1.3	1×10^{9}	4×10^{-2}	4×10^{1}
C	1.5	2×10^{10}	4×10^{-1}	2×10^{1}
D	1.7	5×10^{11}	9×10^{-1}	3×10^{0}
E	2.0	3×10^{12}	1×10^{0}	2×10^{0}

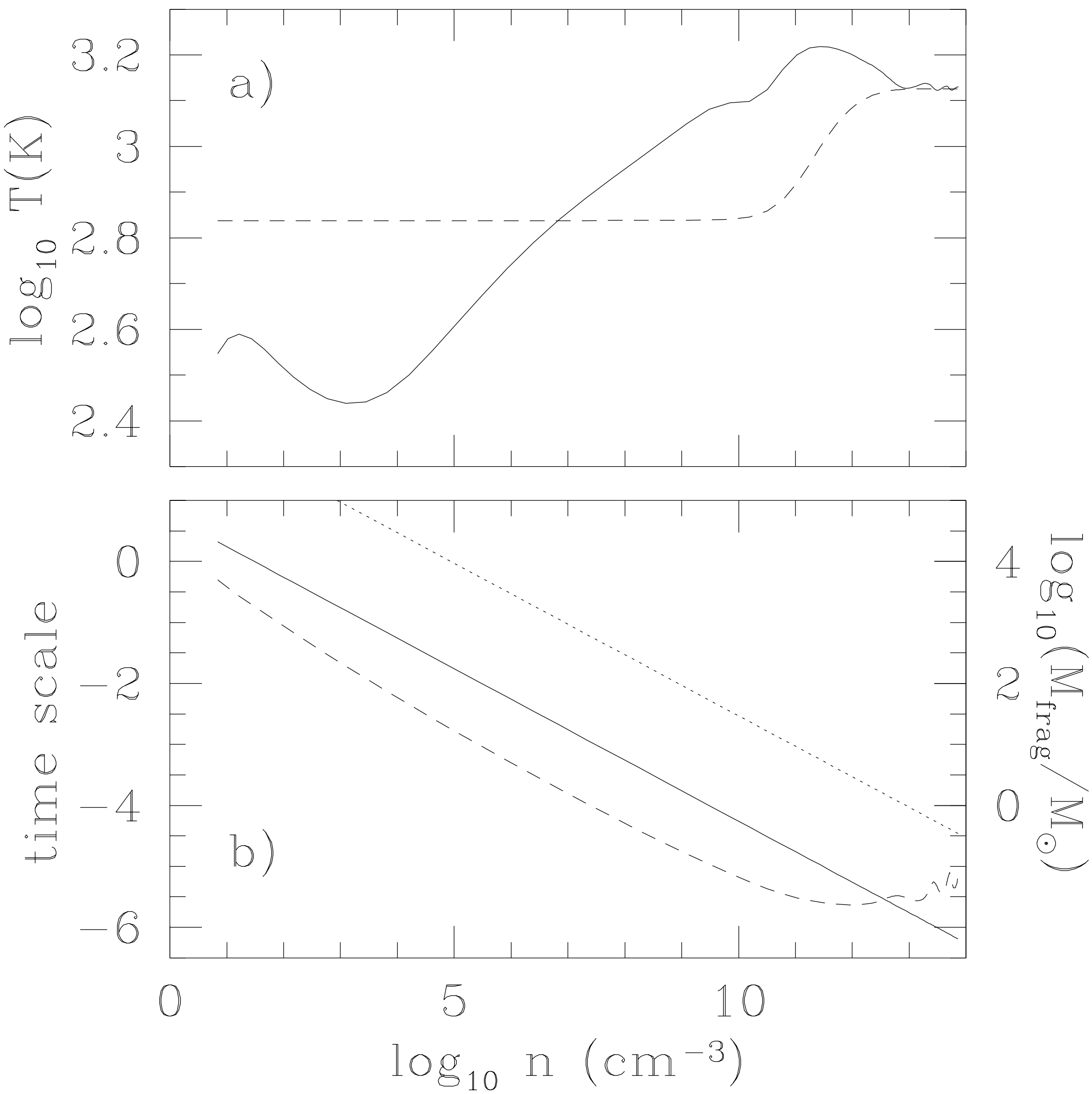

Fig. 1.— a) The thermal evolution of a cylindrical cloud on the n-T plane (solid curve) for case E. The critical temperature is also plotted(dashed curve). b) Evolution of time scales and mass of a fragment for case E. Time scales are normalized by the initial free-fall time. At first the time scale for collapse, t_{dyn}(dashed curve), is shorter than that for fragmentation, t_{frag}(solid curve), by an order of magnitude. But finally t_{frag} becomes shorter than t_{dyn}, at which the collapsing cylinder is thought to fragment and the mass of a fragment(dotted curve) becomes $\sim 2M_{\odot}$.